STUDY ON COMPLEMENTARY GRAPHS AND DEGREE EQUITABILITY

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Abstract

In this paper we present the degree equitability of the graph by defining equitability regularity, equitable connectivity, equitable complete graph and equitable connected graph. In a graph G= (V, E), *Φ* be a function of vertex set V (G) it consists a set of positive integers (0……..n). Then u, v € V (G) be a two vertices and *Φ*-equitable if | *Φ (u)* - *Φ (v)* |≤1. By the degree, adjust equitability between vertices it can be re-define almost all probability of vertices in a graph. Some new categories of graphs that is easily implemented computer networking and equitable coloring is an assignment of colors to the vertices of an un-directed graph.

Keywords: Equitable Path; Equitable Complement Graph; Equitable Domination Number; Equitable Walk; Equitable Line Graph; Equitable Connected Graph; Equitable Regular Graph; Equitable Cut Vertex.

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1. Introduction

All probability graphs we consider here are undirected and finite with multiple edges and no-loops. As usual q=|E| and P=|V| it denoted as edges of a graph G and the number of vertices, respectively. In most commonly we use (X) to denote the sub-graph induced by the set of vertices N (v)and X it denoted as the closed and open neighbourhoods of a vertex (V) respectively. A set of D vertices in a graph G it is a dominate set when it is satisfied the condition of every vertex in V-D is adjacent to some vertex in D. The domination number γ (G) is the minimum cardinality of a dominating set of G. For notations, terminology and detail about parameters of dominations numbers is most important factors in Equitable [1] [2] [3].

A sub-set D of V (G) is called an equitable dominating set of a graph G if for every u € (V-D), there exists a vertex v € D such that u, v € E (G) and |deg (u) – deg (v) |≤1. The minimum cardinality of such a dominating set is denoted by γe (G) and is called equitable domination number of G. In case any vertex u € D then D is minimal, D-{u} is not an equitable dominating set of G. A sub-set S of V is called an equitable-independent set, If for any vertex u € S, v € not Ne (u), for all v € S – {u}. In case a vertex u € V be such that |deg (u) – deg (v)| ≥ 3 for all v € N (u) then u is in each equitable-dominating-set. This type of vertices are called equitable-isolates. The equitable neighbourhood of u denoted by Ne (u) is defined as N_e = {v € V/ v € N (u), |deg (u) – deg (v) |≤ 3} and u € I_e<=>N_e (u) =Ø. The cardinality of N_e (u) is denoted by dege (u) and it is called equitable degree of the vertex u in G. The maximum and minimum equitable degree G of are denoted respectively by Δ_e (G) and δ_e (G). That is by Δ_e (G) =$MAX_{u \in V(G)}$|N_e (u)|,

δ_e = $\min_{ueV(G)}$| N_e (u)|. An edge e=uv is called an equitable edge if |deg (u) – deg (v) |≤ 1.A sunset S of V is called an equitable independent set, if S contains no vertices u, v such that v € N_e (u). If a vertex u € V |deg (u) – deg (v) | ≥2 for all v € N (u) [4].then u is in the equitable dominating set. Such vertices are called equitable isolated vertices.

Let G be a simple graph with n vertices v_1, v_2, ……v_n. then its adjacency matrix A=[a_{ij}] is a n x n matrix whose entries a_{ij} are given by

$$aij = \begin{cases} 1, \text{ if } vi \text{ and } vj \text{ are adjacent} \\ 0, \text{ otherwise} \end{cases}$$

In the same way. The degree equitable adjacent matrix denoted by A_e =[b_{ij}] is a n x n matrix whose entries $b_{ij}$are given by [5]

$$bij = \begin{cases} 1, \text{ if } vi \text{ and } vj \text{ are adjacent} \\ 0, \text{ otherwise} \end{cases}$$

Where the equitable adjacency between any two vertices u, v in V is defined as follows: the vertex v is equitable adjacent to u if and only if u is adjacent to v and also $|deg(u) - deg(v)| \leq 1$.

Social network has been most commonly using application is Degree equitable adjacency. In a network, finite nodes with approximately equal capacity may be interact with each other in a better manner. In society, persons with approximately equal grade,they tend to be friendly. In industry, employees with approximately equal powers form relations and move closely. Equitability among citizens in terms of, health, status, wealth, property etc. is the goal of a democratic nation. These method motivated us to study the degree equitability of a graph by studying and defining some basic properties of degree equitable regularity,degree equitable completeness, and degree equitable connectivity of a graph. Some new relations of graphs and some interesting results are obtained.

2. Elementary Results

In a graph G= (V, E).An equitable tread is defined as a finite alternating sequence of equitable edges andvertices, beginning and ending with vertices. Therefore each equitable edge is incident with the vertices before and after it. No equitable edge appears (is enclosed or pass through) more than once in the equitable tread [6] [7]. A vertex, however, may appear more than once. An equitable tread which begin and ending at the same vertex called closed equitable tread. An equitable tread is not secure if the terminal vertices are dissimilar. An open equitable tread in which no vertex appears more than once is called an equitable pathway. The number of edges in an equitable pathway is called the length of the equitable pathway. A secure equitable tread in which no vertex (except begin and the ending vertex) appears more than once is called an equitable circuit [8].

Now, we want to prove some results is demonstrating the relations between the sum of the equitable degree of the vertices, the number of equitable edges andthe number of edges.

Theorem 2.1. For any graph$G$= *(V, E)* with p vertices v_1, v_2, …… v_p and q edges. $\sum_{i=1}^{p} \deg e\,(vi) \leq 2q$ Further, the equality hold if and only if every edge in *G* is equitable edge.

Proof. We haveN_e (v) ⊊ N (v), for any vertex v in a graph G. Then it is clear that $|N_e (v)| \leq |N (v)|$, and we have:

$$\sum_{i=1}^{p} deg(vi) = \sum_{i=1}^{p} |N\ (vi)| = 2q$$

Similarly,

$$\sum_{i=1}^{p} deg(vi) = \sum_{i=1}^{p} |Ne\ (vi)|$$

Which implies

$$\sum_{i=1}^{p} deg(vi) = \sum_{i=1}^{p} |Ne\ (vi)| \leq \sum_{i=1}^{p} |N\ (vi)|$$

Hence$\sum_{i=1}^{p} \deg e\,(vi) \leq 2q$. Further, if each edge in G is equitable edge, then N_e (v) = N(v) for any vertex v in G that means$\sum_{i=1}^{p} \deg e\,(vi) \leq 2q$. The converse is obvious, if$\sum_{i=1}^{p} \deg e\,(vi) \leq 2q$, then every edge in G is equitable edge.

For any graph G, the number of equitable edge denoted by m_e is called the **equitable size.** The vertex v € V (G) is called **equitable even vertex** (**equitable odd vertex)** if deg_e(v) is even number (odd number) [9].

Theorem 2.2. The sum of the equitable degrees of a graph is twice the number of equitable edge in it that is $\sum_{i=1}^{p} \deg e\,(vi) = 2me$.

Proof. Let G be a graph. If any equitable edge contribution to the equitable degrees of two distinct vertices. Thus when the equitable degrees of the vertices are added. It indicates each equitable edge is counted exactly two times [10]. Then sum of the graph that is equal to$\sum_{i=1}^{p} \deg e\,(vi) = 2me$

Theorem 2.3. Every graph has an even number of equitable odd vertices.

Proof. Suppose that the sum of the equitable degrees ofthe equitable odd vertices is x and the sum of the equitable degrees of the equitable even vertices is y. The number y is even and the number x+ y =2m_e is even. Hence 'x' is even. In case't' equitable odd vertices, then even number x is the sum of 't' odd numbers. So 't' is even.

We can define the equitable comprehensive graph as a connected graph which all its edges are equitable edges and analogous to the equitable complete graph and also we can defined an equitable complement graph of a graph as following:

Definition 2.4. For any graph G, the equitable complement graph of G symbolised by$G^{Ecomp}$is the graph with the same vertices as G and any two vertices u,v are adjacent if u and v are not equitable adjacent in G [11].

The relation between the **equitable complement graph** and the complement of the graph of a graph can be found in the following theorem.

Theorem 2.5. For any graph **G,** $G^{Comp} \subseteq G^{Ecomp}$.

Proof. Let f = UV be any edge in G^{Comp} . Then u and v are not adjacent vertices in G, which implies u and v are not equitable adjacent in G. Hence e = uv is an edge in the graph G^{Ecomp}. Therefore $G^{Comp} \subseteq G^{Ecomp}$.

Theorem 2.6. For any graph G with p vertices, $G^{Ecomp} \approx G^{Comp}$ if and only if G is isomorphic to an equitable complete Graph.

Proof. Let G be an equitable complete Graph and let f =uv be any edge in the graphG^{Ecomp} .then u is non-adjacent to v in G. Hence e=uv is an edge in G^{Comp} and since from the previous theorem we have $G^{Comp} \subseteq G^{Ecomp}$.

Hence$G^{Ecomp} \approx G^{Comp}$. conversely, suppose that $G^{Ecomp} \approx G^{Comp}$ and G is not an equitable complete graph . Then there exists at least one edge e = uv in G which is not equitable which implies that e=uv is an edge in G^{Ecomp} . But clearly u and v are not adjacent inG^{Comp} , a contradiction [12].

A graph G is called an equitable edge-free graph if for any two adjacent vertices u and v in G, |deg (u) – deg (v) | ≥2.

Proposition 2.7. For any graph G with p vertices$G^{Ecomp} \approx K_p$ if and only if G is an equitable edge- free.

Proof. Let G be an equitable edge-free graph with P vertices. Then any two vertices in G^{Ecomp} are adjacent Hence $G^{Ecomp} = K_p$.

Conversely, suppose $G^{Ecomp} = K_p$. Hence any two vertices in G^{Ecomp} are adjacent, then G has no any equitable edge. Therefore G is equitable edge-free [13].

Theorem 2.8. For any graph G with p vertices$G^{Ecomp} \approx K_p^{comp}$ if and only if G isomorphic to the complete graph K_p.

Proof. Let $G \approx K_p$. Then any two vertices u, v € V (G) are equitable adjacent. Hence G^{Ecomp} does not contain any edge. Therefore $G^{Ecomp} \approx K_p^{comp}$.

Conversely, suppose that$G^{Ecomp} \approx K_p^{comp}$, and if possible G is not complete Graph. This implies that there exist at least two non-adjacent vertices u and v in G which are adjacent in G^{Ecomp}. Thus u and v are adjacent in K_p^{comp} , a concentration. Hence G is complete graph K_p.

Corollary 2.9. For any equitable edge-free graph Gwith p vertices,$((G^{Ecomp})^{Ecomp}) \approx G$.

Theorem 2.10. Let *G= (V, E)* be a graph with p vertices and contains at least one non equitable edge with the property that any two vertices u, v in G, we have $N_e [u] \cap N_e [v] = V (G)$. Then $((G^{Ecomp})^{Ecomp}) \approx K_p$.

Proof. Let u,v be any two vertices in G, then we have two cases:

Case 1: if u and v are adjacent vertices in G, then we have two subcases:

- **a)** If uv is equitable edge, then u and v are not adjacent in G^{Ecomp} . Thus u and v are adjacent in $((G^{Ecomp})^{Ecomp})$ [14].
- **b)** If uv is not equitable edge that means $N_e [u] \cap N_e [v] = \Phi$ and since $N_e [u] \cap N_e [v] = V (G)$. Then u and v are adjacent but not equitable adjacent in G^{Ecomp} .Therefore u and v are adjacent in $((G^{Ecomp})^{Ecomp})$.

Case 2: if u and v are non-adjacent vertices in G, then u and v are adjacent vertices in G^{Ecomp} . Since $N_e [u] \cap N_e [v] = V (G)$, we get u and v are not equitable adjacent in G^{Ecomp}. Hence u is adjacent to v in $((G^{Ecomp})^{Ecomp})$. Therefore any two vertices in $((G^{Ecomp})^{Ecomp})$ are adjacent.

Hence $((G^{Ecomp})^{Ecomp}) \approx K_p$.

3. Equitable Connectivity and Equitable Regularity

Definition 3.1. Let G be a graph on n>2 vertices. An equitable disconnecting set of edges is a subset $D \subset E(G)$ such that G-D is equitable disconnected. The edge equitable connectivity, $€_e$ (G), is the smallest number of edges in any equitable disconnecting set.

We adopt the convention that $€_e (K_1) = 0$. Thus $€_e (G) = 0$ if and only if $G = K_1$ or G is equitable disconnected. If $€_e (G) = 1$ then G is a connected graph having an equitable edge f such that G-f is equitable disconnected [15] [16]. An equitable edge whose removal increases the number of equitable component is called **equitable cut-edge** (equitable bridge) of G.

Example. In the graph G in figure 1. The edge (12) is equitable cut edge but not cut edge. So we have $€_e (G) = 1$ but $€ (G) = 2$.

Proposition 3.2. If G is a graph, then $€_e (G) \leq δ_e (G)$. That is, the edge equitable connectivity of G can be no larger than the minimum equitable degree of G.

Proof. Let G be an equitable connected graph on n>2 vertices and suppose $u € V(G)$ is a vertex of equitable degree $deg_e (u) = δ_e (G) > 0$ .since $D = \{uv: v € N_e (u)\}$ is an equitable disconnecting set of edges, $δ_e (G) = |D| \geq €_e (G)$.

The following result is straightforward.

Proposition 3.3. Suppose G is an equitable connected graph. Let e=uv be equitable edge. Then e is an equitable cut-edge of G if and only if P= [u, v] is only equitable path in G from u to v [17].

Definition 3.4. Let G be an equitable connected graph. An equitable vertex cut (or an equitable separating set) of G is a set $S \subset V(G)$ such that G-S is equitable disconnected. The connectivity k_e (G), is the smallest number of vertices in any equitable vertex cut of G. A vertex whose removal increases the number of equitable components of G is called equitable cut-vertex (or point of equitable articulation). For the graph in figure.1 $k_e (G) = 1$ but $k (G) = 2$.

The maximal equitable connected subgraph of G that has no equitable cut-vertex is called an equitable block of G [18] [19].

Theorem 3.5. For any graph G, $k_e (G) \leq €_e (G)$.

Proof. If $€_e (G) = 0$ then G is equitable disconnected and $k_e (G) = €_e (G) = 0$. If $€_e (G) = 1$, then this

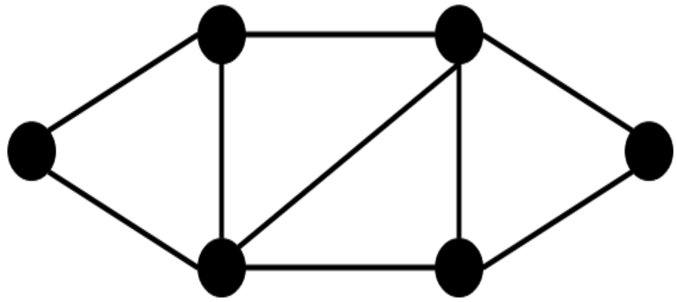

Figure 1. Equitable cut edge.

Graph is equitable connected with equitable bridge *x*, that means $G=K_2$ *or one of the vertices* which incident with x is equitable cut vertex. Therefore, k_e (G) =1. If €$_e$ (G) ≥ 2, then removal €$_e$ (G) edges results in equitable bridge x=uv. For each of these €$_e$(G) -1 edges select an incident vertex different from u or v . The removal of these €$_e$(G) -1 edges select an incident vertex different from u or v. The removal of these €$_e$(G) -1 vertices remove all the €$_e$(G) -1 edges. If the resulting graph is disconnected then k_e (G) ≤ €$_e$(G) . If not, x is equitable bridge of this subgraph and hence the removal of u or v results in an equitable disconnected. Hence, k_e (G) ≤ €$_e$(G) [20][21].

Definition 3.6. A graph G is called k-equitable regular graph if Δ_e (G) = δ_e (G) =k.

Observation 3.10. Every k-regular graph is k-equitable regular graph but the converse is not true, in general.

Example. The graph in the Figure 2 is 2-equitable regular graph but not regular.

4. An Equitable Line Graph and an Equitable Total Graph

Definition4.1. Given a graph*G*, its equitablelinegraph$L_eG$isa graph such that

1)Eachvertexof$L_eG$represents an equitableedge of*G*,and

2)Twoverticesof$L_eG$areadjacentifandonlyif theircorrespondingequitableedgesshareacommon endpoint (are adjacent) in*G*.

Proposition 4.2. The line graph ofequitableconnected graph isconnected.

Proof. If*G*isequitableconnected,itcontainsequitablepathconnectinganytwoofitsedges,whichtranslatesintoapathin$L_eG$containinganytwoofthe vertices ofL_eG. Hence$L_eG$is connected [22] [23].

Observation 4.3. Let*G*be anygraph.Then$L_eG \subseteq LG$.

Remark. Theequitablelinegraphofequitableconnectedgraphisconnectedbutnotequitableconnecte din general. In **Figure 3**,theequitableline graphofequitable connectedgraphisconnectedbutnotequitableconnected.

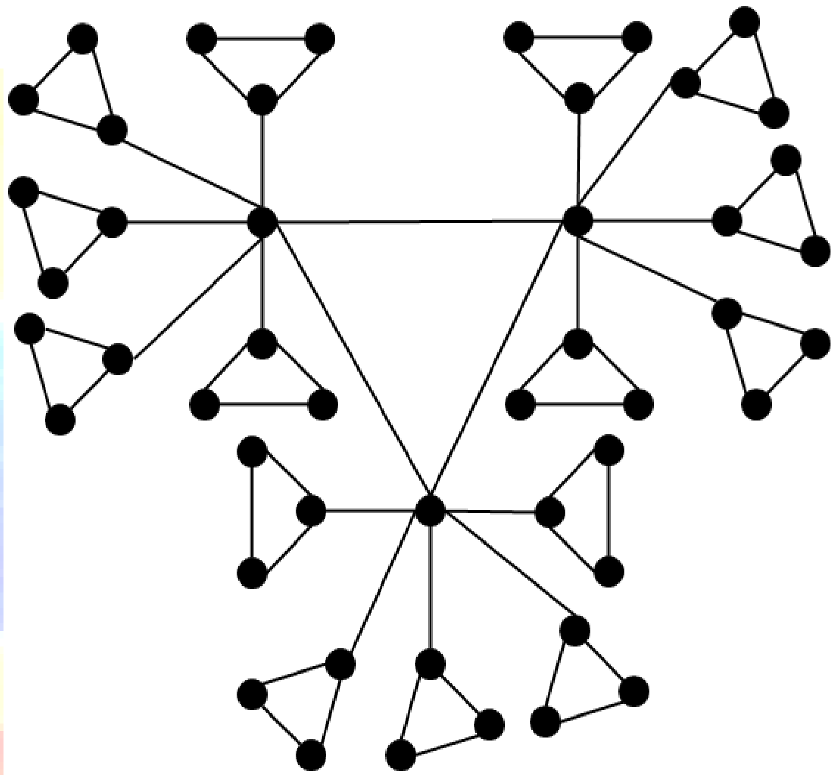

Figure 2. 2-equitable regular graph.

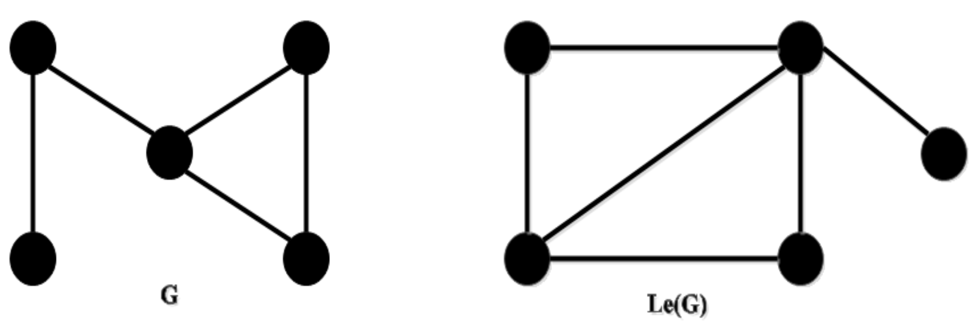

Figure 3. Equitable line graph of equitable connected graph.

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